

Postulates

Invariance of physical laws in all inertial frames.

Constancy of the speed of light c in a vacuum.

Helpful Equations

Lorentz Transformation

Let S denote the frame in which we have an event (spacetime point) with coordinates (t, x, y, z) .

Then let S' denote the frame in which the coordinates of the event are (t', x', y', z') , and which is traveling with a velocity v with respect to S along the x -axis:

$$\begin{aligned}t' &= \gamma \left(t - \frac{vx}{c^2} \right) \\x' &= \gamma(x - vt) \\y' &= y \\z' &= z\end{aligned}$$

γ is the Lorentz factor. Note that as $v \rightarrow 0$, $\gamma \rightarrow 1$ (Galilean limit), and as $v \rightarrow c$, $\gamma \rightarrow \infty$:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Sometimes people write $\beta = v/c$, then $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$.

A sometimes helpful invariant:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

The interval between two events is said to be spacelike if $(\Delta s)^2 < 0$, timelike if $(\Delta s)^2 > 0$ and lightlike if $(\Delta s)^2 = 0$. There can be a cause-effect relationship between two points if and only if the spacetime interval is timelike or (if the event is a photon traveling between two points) lightlike.

Velocity addition

If an object appears to travel at velocity w (parallel to v) in S , then it appears to travel at velocity $w' = \frac{w-v}{1-wv/c^2}$ in S' .

Dynamics

Let m_0 be the rest mass of an object. Then if it is moving at \vec{v} in S , then its energy is:

$$E = \gamma m_0 c^2$$

and its momentum is:

$$\vec{p} = \gamma m_0 \vec{v}$$

Furthermore, $E^2 - |pc|^2 = (m_0 c^2)^2$, which is invariant. Also, $F = \frac{dp}{dt}$ instead of ma .

Time Dilation / Length Contraction

An object has length L in S , and a time interval has length T in S . Then in S' :

$$\begin{aligned} L' &= L/\gamma = L\sqrt{1 - v^2/c^2} \\ T' &= \gamma T = \frac{T}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

The length is contracted and the time is dilated.