Quantum Mechanics GRE Notes

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October 20, 2005

Here is some stuff that you should know about quantum mechanics for the GRE.

- *Schrödinger Equation.* General and time-independent:

$$\begin{aligned} i\hbar\frac{\partial\Psi}{\partial t} &= \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi \\ H\psi &= E\psi \\ \Psi &= \psi e^{-iEt/\hbar} \end{aligned} \tag{1}$$

- *Formalism and Operator Algebra.* This is probably actually the easiest topic for people who have taken 143a. Some equations of use are:

$$\begin{aligned} \langle\alpha|\beta\rangle &= \langle\beta|\alpha\rangle^* \\ \hat{O} &= \hat{O}^\dagger \text{ if } \hat{O} \text{ is Hermitian} \\ \langle\alpha|\hat{O}|\beta\rangle &= \beta\langle\alpha|\beta\rangle \text{ if } |\beta\rangle \text{ is an eigenstate of } \hat{O} \\ &= \int_{-\infty}^{\infty} \mathrm{d}x\, \alpha^*(x)\hat{O}(x)\beta(x) \text{ in 1D} \\ &= A^\dagger \times O \times B \text{ as matrices} \\ [\hat{A},\hat{B}] &= \hat{A}\hat{B} - \hat{B}\hat{A} \\ \therefore\ [\hat{A},\hat{B}] &= -[\hat{B},\hat{A}] \\ [\hat{A}\hat{B},\hat{C}] &= [\hat{A},\hat{C}]\hat{B} + \hat{A}[\hat{B},\hat{C}] \end{aligned} \tag{2}$$

  A Hermitian matrix has real eigenvalues; observables thus must be represented by Hermitian operators.

- *Position and momentum.*

$$\begin{aligned} \hat{p} &= -i\hbar\nabla \\ [\hat{x},\hat{p}] &= i\hbar \\ [\hat{f}(x),\hat{p}] &= i\hbar\frac{\mathrm{d}f}{\mathrm{d}x} \end{aligned} \tag{3}$$

- *De Broglie equation.* This one always trips me up even though it's easy:

$$\begin{aligned} \lambda &= h/p \\ &= h/\sqrt{2mk_BT} \end{aligned} \tag{4}$$

if the particle's energy is thermal. Fun fact: the De Broglie wavelength of thermal neutrons is about 1.78 angstroms.

- *Emitted photons.* Yeah. The name of the game:

$$\begin{aligned} E &= \frac{-\mu e^4}{2h^2}\frac{Z^2}{n^2} \\ &= (-13.61\text{eV})\,\frac{Z^2}{n^2} \text{ for } \mu = m_e \\ \therefore\ \lambda^{-1} &\propto Z^2\left(\frac{1}{n^2}-\frac{1}{n'^2}\right) \end{aligned} \tag{5}$$

This means the key to their beloved positronium problems is that $\mu \to \mu/2$ since the reduced mass is now $(m_e^{-1}+m_e^{-1})^{-1}$. So basically you can do the problem as if it were hydrogen with 13.6 eV $\to$ 6.8 eV.

- *Hydrogen Ground State.* A good thing to know:

$$\psi(r) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a} \tag{6}$$

- *Infinite Square Well.* Straightforward:

$$\begin{aligned} \psi_n &= \frac{1}{\sqrt{a}}\sin\frac{n\pi x}{a} \\ E_n &= \frac{\pi^2 n^2\hbar^2}{2ma^2} \end{aligned} \tag{7}$$

Pay attention to how E_n scales with n. Also, $n \geq 1$ of course.

- *Singlet and triplet states.* These are useful from two perspectives. The first point of view is taking them as eigenstates of particle exchange in a two-particle system. The second is ways of adding two spin-$\frac{1}{2}$ particles to get one spin-1 particle. From the first perspective, the singlet state has eigenvalue -1; from the second, it has $s = m = 0$:

$$|0\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \tag{8}$$

The triplet states have eigenvalue 1 with regards to parity and have $s = 1$ and $m \in \{-1, 0, 1\}$:

$$\begin{aligned} |1\ 1\rangle &= |\uparrow\uparrow\rangle \\ |1\ 0\rangle &= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |1\ -1\rangle &= |\downarrow\downarrow\rangle \end{aligned} \tag{9}$$

- *Selection rules.* They seem to like making you remember these.

$$\begin{aligned} \Delta l &= \pm 1 \\ \Delta m_l &= \pm 1 \text{ or } 0 \\ \Delta j &= \pm 1 \text{ or } 0 \end{aligned} \tag{10}$$

  Here l is the orbital angular momentum; m_l is the z component of l; $j = l + s$ is the total angular momentum. (I'm not exactly sure how you can get $\Delta j = 0$ but I read it on the internet so it must be true!) Typically $\Delta n = \pm 1$ but there is no reason that this must hold. Just remember that the emitted photon is spin 1 and so its angular momentum can be described as one of $|0\ 0\rangle$, $|1\ 1\rangle$, $|1\ 0\rangle$, or $|1\ -1\rangle$.

  The phrase "electric dipole radiation" implies that $\Delta l = 0$ is not physically permissible, even though it is allowed mathematically. Electric dipole transitions are "allowed", while magnetic dipole or electric quadrupole transitions are "forbidden" (but do actually occur).

- *Observed states and expectation values.* The usual format of such a problem is

$$|\psi\rangle = \sqrt{p_1}|k_1\rangle + \sqrt{p_2}|k_2\rangle + \sqrt{p_3}|k_3\rangle + \cdots \tag{11}$$

  The expectation value is $p_1 k_1 + p_2 k_2 + p_3 k_3 + \cdots$ if k_i are the eigenvalues associated with eigenkets $|k_i\rangle$, and the probability of being in a state $|k_i\rangle$ is just p_i. This is all assuming the $|k_i\rangle$ are eigenkets of whatever operator you're finding the expectation value of.

- *Heisenberg relation.* The general version:

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle| \tag{12}$$

  In particular,

$$\begin{aligned} \Delta x \Delta p &\geq \hbar/2 \\ \Delta E \Delta t &\geq \hbar/2 \end{aligned} \tag{13}$$

- *Time Independent First-Order Pertubation Theory.* Given a perturbation H' to some original Hamiltonian with eigenstates $|\psi_n^0\rangle$ and energies E_n^0, the perturbed energies and states are:

$$\begin{aligned} E_n^1 &= E_n^0 + \langle \psi_n^0 | H' | \psi_n^0 \rangle \\ \psi_n^1 &= \psi_n^0 + \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0 \end{aligned} \tag{14}$$

  Recall that

$$\langle \psi_n | \hat{O} | \psi_m \rangle = \int_{-\infty}^{\infty} \psi_n^*(x) \hat{O}(x) \psi_m(x) \mathrm{d}x \tag{15}$$

  in one dimension. (You will often be perturbing the infinite square well.)

- *Fundamental Understanding of the Concepts of Quantum Mechanics.* Yeah. Whatever.