Newton's three laws

1) the concept of equilibrium: things at rest stay at rest unless an external force is applied.
2) **F** = m**a**
3) every action has an equal and opposite reaction

<u>Linear motion:</u>

Galileo's equations of motion for uniform acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Forces

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Special case: Friction. Recall that there is the coefficient of static friction and the coefficient of kinetic friction (coeff. static friction is typically the larger of the two)

$$F_{fric} \leq \mu N$$

Impulse

$$\mathbf{J} = \int \mathbf{F}\, dt = \Delta\mathbf{p}$$

Momentum and energy (**p** conserved always; KE conserved for elastic collisions only)

$$\mathbf{p} = m\mathbf{v}$$

$$K = \frac{1}{2} mv^2 \quad \Delta U_g = mgh$$

Work and Power

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

<u>Rotations</u>

Angular velocity

$\vec{v} = \vec{r} \times \vec{\omega}$

Moment of inertia

$$I = \int r^2 dm = \sum mr^2$$

Principle axes: the axes around which the object can rotate with constant speed, without the need for any torque.

Parallel Axis Theorem (MoI around a new, parallel axis a distance R away from the CM of the object)

$$I_{new} = MR^2 + I^{CM}$$

Angular Momentum

$$\vec{L} = M\vec{R} \times \vec{V} + \sum_i m_i r_i^2$$

Kinetic Energy

$$T = \frac{1}{2}MV^2 + \frac{1}{2}I^{CM}\omega^2$$

Torque

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Galileo’s equations of motion for rotation

$$\omega = \omega_0 + \alpha t$$

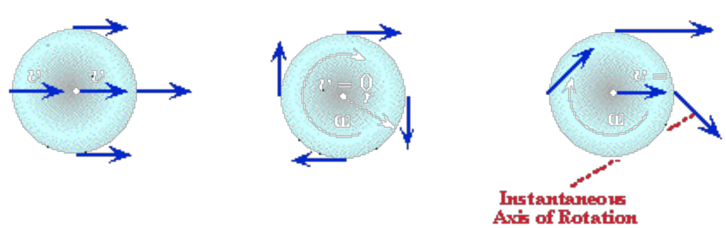
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Special cases:

Uniform circular motion:

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Rolling without slipping: this motion can be considered a linear combination of pure translational and pure rotational motion:



Harmonic motion

Force and energy

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2} kx^2$$

Period

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Period of a mass (*m*) on a spring (*k*)

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Period of a pendulum of length *l* in the Earth's gravity.

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

underdamped, overdamped, and critically damped motion

Gravity, force and potential

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

Central Forces

The effective potential. Note: the first term in the effective potential is the angular momentum barrier that prevents the particle from approaching the source of the gravitational field.

$$V_{eff} = \frac{L^2}{2mr^2} + V(r)$$

$$\frac{1}{2}m\dot{r}^2 + V_{eff} = E$$

Kepler's 2nd and 3rd Laws

$$\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = const$$

$$\tau = A \Big/ \frac{dA}{dt} = 2\pi\sqrt{\frac{m}{k}}a^{3/2}$$

Coriolis Effect

$$F_{coriolis} = -2m(\vec{\omega}\times\vec{v}_{radial})$$

Fluid mechanics:

Pressure on a system due to the fluid above

$$p = p_0 + \rho g h$$

Force of buoyancy; ρ is the mass density of the fluid, V is the volume of fluid displaced.

$$F_{buoy} = \rho V g$$

Motion of a fluid in a tube. A_1 and A_2 are the cross-sectional areas at two places in the tube; v_1 and v_2 are the speeds of the fluid at those places.

$$A_1 v_1 = A_2 v_2$$